7. D. Bharathan and G. Wallis, Int. J. Multiphase Flow, 9, No. 4, 349-366 (1983).
8. J. H. Richter, Heat Transfer in Nuclear Reactor Safety, Seminar, Dubrovnik, Sept. 1-5, 1980 (1982), pp. 343-359.
9. K. Mishima and H. Nishara, Annu. Rep. Res. Reactor Inst. Kyoto Univ., 17, 1-14 (1984).
10. S. Aoki, A. Inoue, M. Aritomi, and Y. Sakamoto, IJHMT, 25, No. 7, 985-990 (1982).
11. M. K. Bezrodnyi and A. I. Beloivan, Inzh. Fiz. Zh., 30, No. 4, 590-597 (1976).
12. D. P. Jordan, Achievements in Heat Transfer [Russian translation], Moscow (1971), pp. 68-143.
13. A. A. Tochigin, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 12-18 (1972).
14. S. S. Kutateladze, I. G. Malenkov, and E. E. Chinnov, Dispersion Systems in Energy Systems [in Russian], Novosibirsk (1982), pp. 3-19.
15. D. A. Labuntsov and Yu.B. Zudin, Trudy MÉI, No. 310, 107-115 (1976).
16. R. Griffith, Teploperedacha, 86, No. 3, 36-44 (1964).
17. A. N. Kovalev, Inzh. Fiz. Zh., 53, No. 4, 557-560 (1987).

THE EFFECT OF THE NONSTEADY STATE AND TURBULENCE ON INTERPHASE HEAT AND MASS TRANSFER IN THE RELATIVE MOTION OF BUBBLES IN A BOILING STREAM

V. A. Gerliga and V. I. Skalozubov

UDC 536.421.3

An analytical solution is proposed for the heat flow between vapor bubbles and a liquid with consideration of the nonsteady relative velocity of bubble motion in the nonsteady pressure field of a boiling stream.

Numerous papers have been devoted to the questions of rates of change in bubble dimensions in steady-state nonmoving volumes of a boiling liquid; reviews of these papers are presented, for example, in [1, 2]. However, to create closed mathematical models of nonequilibrium boiling flows we must have relationships which describe the transfer of heat and mass between moving vapor bubbles and a liquid, with consideration given to the definitive features of bubble evolution within the stream. At the present time, only individual special cases have been investigated.

A solution was obtained in [3] for the specific heat flow q between the vapor bubbles and a liquid, with consideration given to the relative nonsteady velocity of phase motion, while the quasisteady self-similar numerical approximation of that solution is presented in [4]. In [5] and in the works of Nakoryakov et al. [6] analytical solutions were obtained for q with consideration of the nonsteady nature of the pressure field in the process of bubble growth in the absence of any effect exerted by the induced convection that is due to the relative motion of the bubbles.

Semiempirical relationships have been derived in [7-9] and in these allowance is made for the decisive effect of turbulence on the specific heat flow between vapor bubbles and the liquid in a stream. These relationships in this case make no allowance for the relative motion of the bubbles, nor of the nonsteady nature of the flow parameters, and in the limit (an insignificant degree of flow turbulence) these relationships do not correspond to other special solutions.

In the general case we have the combined effect of all of the above-enumerated factors on the exchange of heat and mass between bubbles and liquid in a boiling stream. However, at the present time there exists no solution which allows for the nonsteady nature of the relative velocity of bubble motion in a field of nonsteady pressure for a boiling turbulent stream.

Odessa Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 56, No. 2, pp. 200-206, February, 1989. Original article submitted September 1, 1987.

Let us examine the problem of determining the exchange of heat and mass between vapor bubbles and a liquid in the case of a nonsteady relative velocity $U(\tau)$ of bubble motion and a prehistory of bubble development in a field of boiling-stream nonsteady parameters. We will solve this problem under the following basic assumptions: 1) the exchange of heat and mass between bubbles and the liquid is governed by the thermal resistance at the boundary of phase separation (from an energy standpoint the thermal pattern matches the Labuntsov classification); 2) the bubble is spherical in shape and has a radius R; 3) the liquid is incompressible; 4) the motion of the liquid about the bubble surface is potential; 5) we will neglect bubble interaction.

The density of the heat flow between the bubble and the liquid is defined as follows:

$$q = -\frac{\lambda_L}{\pi}\int_0^{\pi}\frac{\partial t}{\partial y}(y=0)\sin\theta d\theta. \tag{1}$$

We thus have to know the field of temperature distribution around the bubble. On the basis of the adopted assumptions, the equation for the energy of the liquid around the surface of the bubble [10]:

$$\frac{\partial t}{\partial \tau} + V_r\frac{\partial t}{\partial r} + \frac{V_\theta \partial t}{r\partial\theta} = a_L\frac{\partial^2 t}{\partial r^2} + \frac{2a_L}{r}\frac{\partial t}{\partial r} + \frac{a_L}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right), \tag{2}$$

where

$$V_r = -U\left(1-\frac{R^3}{r^3}\right)\cos\theta + \frac{R^2}{r^2}W_R;\ W_R = \frac{q}{l\rho_v} - \frac{R}{3C_V^2}\frac{dP}{d\tau}; \tag{3}$$

$$V_\theta = U\left(1+\frac{R^3}{2r^3}\right)\sin\theta. \tag{4}$$

A more complete expression for V_r and V_θ with consideration of the interaction between the bubbles is presented in [4]. As was demonstrated in [10, 11], system (2)-(4) with a thickness of the thermal boundary layer very much smaller than the bubble dimension R can be reduced to the equation

$$\frac{\partial T}{\partial x} - \frac{R_0^2}{a_L}\left[3\frac{U}{R}\cos\theta + \frac{2W_R}{R}\right]Y\frac{\partial T}{\partial Y} + \frac{3}{2}\frac{R_0^2}{a_L}\frac{U}{R}\sin\theta\frac{\partial T}{\partial\theta} = \frac{\partial^2 T}{\partial Y^2}, \tag{5}$$

where

$$T = \frac{t-t_\infty}{t_\infty};\ Y = \frac{y}{R_0};\ y = r - R;\ x = \frac{a\tau}{R_0^2}.$$

The initial and boundary conditions will be taken in the form:

$$T(Y,\ 0,\ \theta) = 0, \tag{6}$$

$$T(0,\ x,\ \theta) = T_0(x), \tag{7}$$

$$T(\infty,\ x,\ \theta) = 0. \tag{8}$$

We will solve the formulated problem by the method of integral transformations. We will use the Fourier sine transform:

$$T_w(x,\ \theta) = \int_0^{\infty} T(V,\ x,\ \theta)\sin wY dY, \tag{9}$$

$$T(Y,\ x,\ \theta)=\frac{2}{\pi}\int_0^\infty T_w(x,\ \theta)\sin wY\,dw. \tag{10}$$

With consideration of conditions (8) and (7), from expressions (5) and (6) we obtain:

$$\frac{\partial T_w}{\partial x}+[U_1\cos\theta+U_2]\,w\,\frac{\partial T_w}{\partial w}+\frac{U_1}{2}\sin\theta\,\frac{\partial T_w}{\partial\theta}=wT_0-w^2T_w-(U_1\cos\theta+U_2)\,T_w, \tag{11}$$

where $T_w(0,\ \theta)=0$,

$$U_1=\frac{3}{2}\,R_0^2U/aR;\quad U_2=2R_0^2W_R/aR. \tag{12}$$

The equations of the characteristics for (11) have the form

$$\frac{d\theta}{\sin\theta}=\frac{U_1}{2}\,dx, \tag{13}$$

$$\frac{dw}{w}=(U_1\cos\theta+U_2)\,dx, \tag{14}$$

$$\frac{dT_w}{dx}+(U_1\cos\theta+U_2+w^2)\,T_w=wT_0. \tag{15}$$

After integration of (13)-(15) we will obtain, respectively:

$$\operatorname{tg}\frac{\theta}{2}=\exp\left\{\int_0^x\frac{U_1}{2}\,d\sigma-C_1\right\}, \tag{16}$$

$$w=\exp\left\{\int_0^x\left[U_1\,\frac{1-\exp\left\{2\left[\int_0^x\frac{U_1}{2}\,d\sigma-C_1\right]\right\}}{1+\exp\left\{2\left[\int_0^x\frac{U_1}{2}\,d\sigma-C_1\right]\right\}}+U_2\right]dt-C_2\right\}, \tag{17}$$

$$T_w=\exp\left[-\int_0^x(U_1\cos\theta+U_2+w^2)\,ds\right]\left\{\int_0^x wT_0\,\exp\left[\int_0^\xi(U_1\cos\theta+U_2+w^2)\,d\sigma\right]d\xi+C_3\right\}. \tag{18}$$

We will determine the constants C_1, C_2, C_3 from the expression $C_3=\Phi(C_1,\ C_2)$ in a manner analogous to that described in [3, 10]. We will express C_1, C_2, C_3, respectively, in accordance with (16), (17), and (18). With consideration of condition (12), after rather cumbersome transformations, following [10], for the transform $T_w(x,\ \theta)$ we will obtain the expression:

$$\begin{aligned}T_w(x,\ \theta)=\int_0^x T_0w\exp\{-2\,[\varphi(x)-\varphi(\xi)]\}\times\\ \times\exp\left\{-w^2\int_\xi^x\exp[-2\varphi(x)+2\varphi(s)]\,ds\right\}d\xi,\end{aligned} \tag{19}$$

$$\varphi(b)-\varphi(a)=\int_a^b\left[U_1\,\frac{1-A(s,\ x)\operatorname{tg}^2\dfrac{\theta}{2}}{1+A(s,\ x)\operatorname{tg}^2\dfrac{\theta}{2}}+U_2\right]ds, \tag{20}$$

$$A(s, x) = \exp\left(-\int_s^x U_1 d\sigma\right). \tag{21}$$

We will restore the original $T(Y, x, \theta)$ from expressions (10) and (19):

$$T = \frac{1}{2\sqrt{\pi}} Y \int_0^x T_0 \exp\{2[\varphi(\xi) - \varphi(x)]\} [F(\xi, x)]^{-\frac{3}{2}} \exp\left\{-\frac{Y^2}{4F(\xi, x)}\right\} d\xi, \tag{22}$$

where

$$F(\xi, x) = \int_\xi^x \exp[2\varphi(s) - 2\varphi(x)]\, ds.$$

Having brought (22) to dimensional form, and after substitution into Eq. (1), we will finally obtain:

$$q = \frac{\lambda_L}{2\pi^{\frac{3}{2}}\sqrt{a}} \int_0^\pi \sin\theta \int_0^\tau (t_\infty - t_0) \exp\{2[\varphi(\xi) - \varphi(\tau)]\} [F(\xi, \tau)]^{-3/2} d\theta d\xi. \tag{23}$$

Expression (23) characterizes the interphase specific heat flow between the vapor bubbles and the liquid in nonsteady relative bubble motion in a field of nonsteady boiling-flow pressure.

Numerical analysis of the derived solution demonstrates that on satisfaction of the relationships:

$$\mathrm{Pe}_R = \frac{2UR}{a} \ll \mathrm{Ja} = \frac{C_L \rho_L (t_\infty - t_0)}{l\rho_v}; \ |t_0(\tau = 0) - t_\infty| \gg \frac{dt_0(\tau)}{d\tau}\tau \tag{24}$$

from expression (23) follow the results that correspond to the familiar quasisteady approximation of the Scriven solution, obtained by Labuntsov and his associates:

$$\mathrm{Nu} = \frac{2qR}{\lambda_L(t_\infty - t_0)} = \frac{12}{\pi}\mathrm{Ja}\left[1 + \frac{1}{2}\left(\frac{\pi}{6\,\mathrm{Ja}}\right)^{\frac{2}{3}} + \frac{\pi}{6\,\mathrm{Ja}}\right]. \tag{25}$$

The first condition in (24) indicates that the velocity of the relative bubble motion is rather small and the velocity field about the bubble surface is determined by the convection that is a consequence of the motion of the boundary of phase separation, in accordance with formulas (3) and (4). The second condition in (24) indicates that the rate at which the pressure is reduced (the temperature of the vapor) is rather small during the course of the process relative to the temperature difference at the initial instant of time, and a change in the difference $(t_0 - t_\infty)$ during the time τ is, therefore, insignificant.

When the conditions

$$|t_0(\tau = 0) - t_\infty| \gg \frac{dt_0(\tau)}{d\tau}\tau; \ \mathrm{Pe}_R(\tau = 0) \gg \frac{d\mathrm{Pe}_R(\tau)}{d\tau}\tau \tag{26}$$

are met, solution (23) corresponds to the quasisteady self-similar approximation of the Ruckenstein solution obtained in [4]:

$$\mathrm{Nu} = \frac{12}{\pi}\mathrm{Ja}\left[1 + \frac{1}{2}\left(\frac{\pi}{6\,\mathrm{Ja}}\right)^{\frac{2}{3}} + \frac{\pi}{6\,\mathrm{Ja}}\right] + 1.13\,\mathrm{Pe}_R^2 \times \\ \times \left[\frac{1}{13\,\mathrm{Ja}^{3.3} + \mathrm{Pe}_R^{1.3}} - \frac{6\,\mathrm{Ja}^{0.63}}{31\,\mathrm{Ja}^{4.3} + \mathrm{Pe}_R^2}\right]. \tag{27}$$

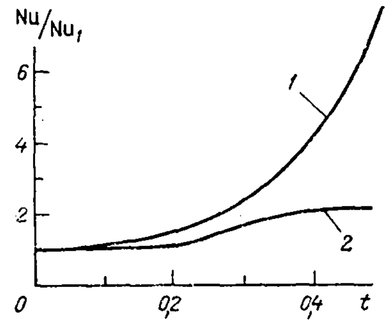

Fig. 1. Ratios of the Nu number determined from (23) to the Nu_1 number determined from (27) for fixed Ja numbers and for various laws governing the relative velocity of bubble motion (P = 2.0 MPa; $R_0 = 10^{-4}$ m): 1) Ja = 10, $Pe_R = 10^3 \tau$; 2) Ja = 10, $Pe_R = 10 + 0.1 \tau$. t, sec.

Conditions (26) indicate that we can neglect the changes in the temperature and velocity differences relative to bubble motion insofar as these relate to the values at the initial instant of time. Thus, in satisfying conditions (26) the quasisteady approximation is acceptable; in fact, that is precisely the role of formula (27).

From a comparison of calculations involving (23) and (27) we can draw the following conclusions:

1. When $|Pe_R(\tau)| \gg |Pe_R(\tau = 0)|$ the differences may be significant. The Nu number ratio determined from (23) relative to Nu_1 determined from (27) reaches values of 5-10 (see Fig. 1).

2. With a reduction in the ratio $Pe_R/Pe_R(\tau = 0)$ the differences are sharply diminished (see Fig. 1). In this case, deviations in numerical calculations may arise by the self-similarity of solution (27), i.e., by the fact that according to (27) Nu is independent of the initial conditions.

3. According to solution (27), the relative velocity of bubble motion (Pe_R) influences the Nu number in the region of high temperature differences (Ja $\gg$ 1) when $Pe_R > 10$Ja. Numerical analysis of (23) demonstrates that the effect of the relative velocity of bubble motion on q becomes significant even when $Pe_R^2 > 10$Ja. Tokuda, Yang, and Clark [12] reached a similar conclusion on the basis of numerical analysis.

On satisfaction of the conditions

$$Pe_R \approx 0;\ \mathrm{Ja} \gg 1 \tag{28}$$

solution (23) corresponds to results obtained from the familiar formula [5]:

$$q = \lambda_L \sqrt{\frac{3}{\pi a}} \left[\frac{(t_\infty - t_0(\tau = 0))}{\sqrt{\tau}} + \int_0^\tau \frac{\partial}{\partial \xi} (t_\infty - t_0) \frac{d\xi}{\sqrt{\tau - \xi}} \right]. \tag{29}$$

The first condition in (28) indicates that the forced convection caused by the relative motion of the bubble exerts no influence on the flow of heat between the bubbles and the liquid. The region of applicability for solution (29) in the second of the conditions in (28) is also confirmed by numerical calculations [13].

Unfortunately, at the present time there are no experimental data sufficiently adequate to validate the obtained solution (23). However, the cited special solutions (25), (27), and (29), which at the limit correspond to (23), were confirmed experimentally under corresponding conditions of applicability.

It should be pointed out that the initial condition (6) that is usually employed has not been formulated very rigorously [1]. Indeed, at the initial instant of time in the general case: $t(y, 0, \theta) \neq t_\infty$. Let us examine problem (5) for the following initial and boundary conditions:

$$T(Y,\ 0,\ \theta)=f(Y,\ \theta);\ T(0,\ x,\ \theta)=T_0(x);\ T(\infty,\ x,\ \theta)=0. \tag{30}$$

In this case, system (5), (30) after the Fourier sine transform (9) can be presented in the form

$$\frac{\partial T_w}{\partial x}+[U_1\cos\theta+U_2]\,w\frac{\partial T_w}{\partial w}+\frac{U_1}{2}\sin\theta\,\frac{\partial T}{\partial\theta}= \tag{31}$$
$$=wT_0-(w^2+U_1\cos\theta+U_2)\,T_w,$$

$$T_w(x,\ \theta)|_{x=0}=\int_0^\infty f(Y,\ \theta)\sin wY\,dY=f_w(\theta). \tag{32}$$

Using Eqs. (31) and (32) for transformations similar to (13)-(18), for the transform $T_w(x,\ \theta)$ we will obtain the following expression:

$$T_w(x,\ \theta)=f_w\exp[\varphi(0)-\varphi(x)]\exp\left\{w^2\int_0^x\exp[\varphi(s)-\varphi(x)]\,ds\right\}+$$
$$+\exp[\varphi(0)-\varphi(x)]\exp\left\{-w^2\int_0^x\exp[\varphi(s)-\varphi(x)]\right\}w\int_0^x T_0\exp[2\varphi(\xi)- \tag{33}$$
$$-\varphi(x)-\varphi(0)]\exp\left\{w^2\int_0^\xi\exp[2\varphi(\sigma)-2\varphi(x)]\,d\sigma\right\}d\xi,$$

where the quantity φ corresponds to (20). Using formula (10) to restore the original $T(\varphi,\ x,\ \theta)$ from (33), after rather cumbersome transformations, following [14], we find

$$q=\frac{\lambda_L\sqrt{\pi}}{2\pi^2}a^{-\frac{1}{2}}\exp[\varphi(0)-\varphi(\tau)]\int_0^\pi\sin\theta\,[B(\tau)]^{-\frac{1}{2}}\times$$
$$\times\int_0^\infty t_\infty f(z)\,z\exp\left[-\frac{z^2}{4aB(\tau)}\right]dz\,d\theta+q\,[f(Y)=0], \tag{34}$$

where $q[f(Y) = 0]$ is determined from solution (23) derived above. In expression (34)

$$B(\tau)=\int_0^\tau[\varphi(s)-\varphi(\tau)]\,ds. \tag{35}$$

Thus, for a known initial distribution of liquid temperatures $f(y)$ the specific heat flow q is determined under the conditions employed here from formula (34). A simplified numerical analysis shows that the first term in (34) diminishes rapidly with increasing τ.

To account for the effect of turbulence on interphase heat and mass exchange, it is proposed in [11] within the framework of a phenomenological approach formally to present the coefficient of thermal conductivity a in the form of an additive sum of molecular a_L and a semiempirical turbulent coefficient a_T that is a function of the flow regime parameters:

$$a=a_L\left[1+k\left(1-\varphi\frac{1-\lambda_1}{1+\lambda_1}\right)\mathrm{Pe}\,\delta\right], \tag{36}$$

where according to [15]: $\lambda_1 = W_0R^2/9\nu_LD$. As was demonstrated in [11], the turbulent mechanisms of transfer may affect the heat and mass exchange between the bubbles and the liquid where the dimensions of the bubbles are greater than the internal scale of the turbulence $\lambda_0 \sim (\nu^0D/W_0{}^3)^{0.25}$. Consequently,

$$\delta=\begin{cases}1, \text{ when } R>\lambda_0,\\ 0, \text{ when } R\leqslant\lambda_0.\end{cases} \tag{37}$$

Comparison with the experimental data of Kevorkov [8] shows that the coefficient $k = 3.3 \cdot 10^{-6}$. The proposed approach of describing the effect of turbulence on the flow of heat between the vapor bubbles and the liquid, unlike the formulas derived in [7-9], makes allowance also for the combined effect of the nonsteady relative velocity of bubble motion in the nonsteady pressure field. It should be noted that the proposed allowance for liquid turbulence is approximate and requires additional work.

Thus, having determined the coefficient a in (23) and (34) in the form of (36) and (37), we obtain expressions for a description of the specific flow of heat between the vapor bubbles and the liquid in the case of nonsteady relative motion in the field of nonsteady flow pressure, with consideration given to the influence exerted by the turbulence mechanisms of transfer.

NOTATION

λ_L, a_L, ν_L, C_L, are, respectively, the coefficients of thermal conductivity, heat conduction, viscosity, and heat capacity of the liquid; t, temperature of the liquid; r, θ, spherical coordinates; R_0, R, initial and instantaneous radius of the bubble; ℓ, latent heat of vapor formation; C_V, speed of sound in the vapor; τ, time; P, pressure; ρ_V, ρ_L, densities in the vapor and in the liquid; $Fo = a\tau/R_0^2$, Fourier number; φ, true volumetric vapor content in formula (36), a function; $Pe = W_0 D/a_L$, Peclet number; W_0, average velocity of the stream; D, diameter of the channel.

LITERATURE CITED

1. V. I. Tolubinskii, Heat Exchange in Boiling [in Russian], Kiev (1980).
2. V. F. Prisnyakov, The Physical Theory of Liquid Boiling [in Russian], Dnepropetrovsk (1977).
3. E. Ruckenstein and J. Davis, Intern. J. Mass Transfer, 14, 939-952 (1971).
4. A. E. Kroshilin, V. E. Kroshilin, and B. I. Nigmatulin, Teplofiz. Vys. Temp., 22, No. 2, 355-362 (1984).
5. Jones and Zuber, Teploperedacha, No. 3, 75-79 (1981).
6. V. E. Nakoryakov and I. R. Shreiber, Teplofiz. Vys. Temp., 17, No. 4, 798-803 (1979).
7. D. A. Labuntsov and R. I. Soziev, Heat and Mass Transfer [in Russian], Vol. 2, Part 1, Minsk (1972), pp. 453-456.
8. L. R. Kevorkov, "Investigating the effect of water temperature on the mechanism of cavitational phenomena," Doctoral Dissertation, Moscow (1975).
9. A. A. Avdeev, Teploenergetika, No. 1, 53-56 (1986).
10. V. A. Gerliga and V. I. Skalozubov, "Rates of change in bubble dimensions in a field of variable stream parameters," Ukr. NIINTI, No. 2303 (1986).
11. V. I. Skalozubov, "Flow of adiabatically bubbling streams in conduits," Doctoral Dissertation, Odessa (1986).
12. N. Tokuda, J. Clar, and W. Yang, Proc. 4th Int. Mass Transfer Conference, Paris, 1970 (Preprint B 7.5).
13. A. E. Kroshilin, V. E. Kroshilin, and B. I. Nigmatulin, Teplofiz. Vys. Temp., 24, No. 3, 533-538 (1986).
14. V. A. Gerliga and V. I. Skalozubov, "The effect of nonsteady liquid parameters on the rate of change in bubble dimensions," Ukr. NIINTI, No. 135 (1986).
15. A. V. Gorin, Hydrodynamics and Heat Exchange [in Russian], Novosibirsk (1979), pp. 115-121.